Company: HSBC
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Subject: Convexity adjustment on terminal rate models - Replication of CMS product and Cash-settled Options

The convexity adjustment phenomenon takes place in interest rates derivatives when a product involves the markets rate payment, observed in the future, at a different date than its natural payment date. The typical case is the CMS, where a swap rate, usually paid several times at regular due dates, is paid in one time, in a Libor rate way.

Mathematically, the valuation of such products involves a numeraire change, from the "natural" numeraire N (typically the swap annuity) under which the terminal distribution of the underlying rate S is known, to the "payment numeraire" P (typically a zero coupon) by the formula:

$$price(t) = P(t, T_p)\ \mathrm{E}^P \left[\text{payoff}[S(T_f)] \right] = N(t)\ \mathrm{E}^N \left[\text{payoff}[S(T_f)]\ \mathrm{E}\left[\frac{P(T_f, T_p)}{N(T_f)} \middle| S(T_f) = S \right] \right]$$

If the right side conditional expectation called the numeraire change function $Z_p(S)$ is known, then the price is obtained by simple replication by "natural" options (swaptions) on the underlying rate R: it then deduced from the smile only (therefore the "terminal model" name). Unfortunately in the great majority of cases, this expectation is not known. We then get the choice between: using a complete market model to reconstitute the expectation, which is heavy and not much reliable and staying in "terminal model", arbitrarily imposing the shape of the numeraire change, which generates a model risk. This function has not been thoroughly studied in literature, despite the fact is used in many derived products: one is usually satisfied with more or less justified approximations. But within a perturbed high volatility market, it becomes necessary to precisely re-study the foundations of our valuation tools, in order to precisely identify the hidden risks of used models.

During my internship, I have studied five numeraire change models. Two of them were proposed by the front office and the three others are the result of my bibliographic research. Those functions must satisfy some non-arbitrage constraints. Those constraints have been studied. I have proposed three new numeraire changes that satisfy those constraints and are as flexible as possible for model risk. As those functions do not satisfy the constraint of the good expectation, a study of two expectations adjustment methods was made. Finally I conducted a study on three smiles in order to see if they have an impact on the replication formula.

In the programming area I have made a new pricer by replication. This pricer is a very general pricer. For a given payoff, a given numeraire change and a given smile and with some mathematical tools we are able to price any product on CMS or cash-settled swaption. This pricer can easily be used from others as one can add new payoff functions, new numeraire change functions and new smiles. The good point is that we can also use it for others replication formulae as one can add another method to compute the expectation.

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