ERRATUM

Samir Salem pointed out an error in the proof point (i) of Lemma C.2 of [1]: we applied the Jensen inequality to the Lebesgue measure on \((\mathbb{R}^3)^{N-k}\), which is not a probability measure. And by the way, we did not provide any rigorous argument ensuring that \(\int_{(\mathbb{R}^3)^{N-k}} \nabla v_1 F(v_k^N, v_{N-k}^N) \, dv_{N-k}^N\), which appears in argument of \(\Psi_r\), is well defined almost everywhere. We can justify the use of the Jensen inequality by saying that \(\Psi_r\) is actually homogeneous of degree one. That would allow to apply the Jensen inequality with respect to the uniform probability of a ball of given radius, and then obtain the requested inequality by passing to the limit. But passing to the limit requires a proper answer to the second point.

Here is a more direct proof that fixes both issues. Since \(F\) is symmetric, we can write

\[
I^r(F) = \int_{(\mathbb{R}^3)^N} \frac{\nabla v_1 F(v^N)}{|F(v^N)|^{2r-1}} \, dv^N \quad \text{and} \quad I_1(F) = \int_{(\mathbb{R}^3)^N} \langle v_1 \rangle^\gamma \frac{\nabla v_1 F(v^N)^2}{F(v^N)}.
\]

Fix \(k \in \{1, \ldots, N-1\}\) and set \(v_k^N = (v_1, \ldots, v_k)\) and \(v_{N-k}^N = (v_{k+1}, \ldots, v_N)\). Since \(2r > 1\), we have, by the Hölder inequality,

\[
\int_{(\mathbb{R}^3)^{N-k}} |\nabla v_1 F(v_k^N, v_{N-k}^N)| \, dv_{N-k}^N \leq \left( \int_{(\mathbb{R}^3)^{N-k}} \frac{|\nabla v_1 F(v_k^N, v_{N-k}^N)|^{2r}}{|F(v_k^N, v_{N-k}^N)|^{2r-1}} \, dv_{N-k}^N \right)^{\frac{1}{2r}} \left( \int_{(\mathbb{R}^3)^{N-k}} |F(v_k^N, v_{N-k}^N)|^{\frac{2r-1}{2r}} \, dv_{N-k}^N \right)^{\frac{1}{2r}}.
\]

The two integrals in the r.h.s. are finite almost everywhere, since \(F\) is a probability density and \(I^r(F) < \infty\). Consequently,

\[
\frac{|\nabla v_1 F_k(v_k^N)|^{2r}}{|F_k(v_k^N)|^{2r-1}} \leq \int_{(\mathbb{R}^3)^{N-k}} \frac{|\nabla v_1 F(v_k^N, v_{N-k}^N)|^{2r}}{|F(v_k^N, v_{N-k}^N)|^{2r-1}} \, dv_{N-k}^N.
\]

Integrating this inequality in \(v_k^N\), we obtain \(I^r(F_k) \leq I^r(F)\). Choosing next \(r = 1\), multiplying the inequality by \(\langle v_1 \rangle^\gamma\) and integrating in \(v_k^N\), we obtain \(I_1(F_k) \leq I_1(F)\).

REFERENCES