Adaptive Boxcar Deconvolution on Full Lebesgue Measure Sets*

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Abstract

We consider the non-parametric estimation of a function that is observed in white noise after convolution with a boxcar, the indicator of an interval $(-a, a)$. In a recent paper **?** have developed a wavelet deconvolution method (called `WaveD`) that can be used for "certain" boxcar kernels. For example, `WaveD` can be tuned to achieve near optimal rates over Besov spaces when a is a Badly Approximable (BA) irrational number. While the set of all BA's contains quadratic irrationals e.g. $a = \sqrt{5}$ it has Lebesgue measure zero, however. In this paper we derive two tuning scenarios of `WaveD` that are valid for "almost all" boxcar convolution (i.e. when $a \in A$ where A is a full Lebesgue measure set). We propose (i) a tuning inspired from Minimax theory over Besov spaces; (ii) a tuning derived from Maxiset theory providing similar rates as for WaveD in the BA widths setting. Asymptotic theory informs that (i) in the worst case scenario, departures from the BA assumption affect `WaveD` convergence rates, at most, by log factors; (ii) the Maxiset tuning, which yields smaller thresholds, is superior to the Minimax tuning over a whole range of Besov sub-scales. Our asymptotic results are illustrated in an extensive simulation of boxcar convolution observed in white noise.

1 Introduction

Suppose we observe the stochastic process

$$Y_n(dt) = f \star b(t)dt + \sigma n^{-1/2} W(dt), \quad t \in T = [0,1], \tag{1}$$

where $b(t) = \frac{1}{2a}\mathbb{I}\{|t| \leq a\}$, σ is a positive constant, $W(.)$ is a Gaussian white noise and

$$f \star b(t) = \frac{1}{2a}\int_{-a}^{a} f(t-u)du\,. \tag{2}$$

Our goal is to recover the function f from the noisy blurred observations (**??**). The boxcar half-width a in (**??**) is assumed to be known. Further, we assume that the function f is periodic on the unit interval T. This is the so-called boxcar deconvolution problem. This is

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