Passive imaging using cross correlations of ambient noise signals

Josselin Garnier
Laboratoire de Probabilités et Modèles Aléatoires
& Laboratoire Jacques-Louis Lions,
Université Paris 7, 2 Place Jussieu, 75251 Paris Cedex 5, France
Email: garnier@math.jussieu.fr

George Papanicolaou
Mathematics Department,
Stanford University, Stanford, CA 94305
Email: papanicolaou@stanford.edu

***Abstract*—It is possible to estimate the travel time or even the full Green's function between two passive sensors from the cross correlation of recorded signal amplitudes generated by ambient noise sources. Using the stationary phase method we show that it is possible to image reflectors buried in a smoothly varying medium by migrating the cross correlations of the noise signals.**

I. Introduction

It is possible to estimate the Green's function of the wave equation in an inhomogeneous medium by cross correlating the signals emitted by ambient noise sources and recorded by a passive sensor array [1], [7], [14], [15]. This idea can be used for travel time estimation and background velocity estimation [9], [11], and also for passive sensor imaging of reflectors [9], [8], which consists in backpropagating or migrating the cross correlations of the recorded signals. In this paper we analyze the properties of the cross correlations in the presence of reflectors and build migration functionals for their localization.

II. Imaging by cross correlation of noisy signals

A. The wave equation with noise sources

We consider the solution u of the wave equation in a three-dimensional inhomogeneous medium with propagation speed $c(\boldsymbol{x})$:

$$\frac{1}{c^2(\boldsymbol{x})}\frac{\partial^2 u}{\partial t^2} - \Delta_{\boldsymbol{x}} u = n(t,\boldsymbol{x}). \tag{1}$$

The term $n(t,\boldsymbol{x})$ models a random distribution of noise sources. It is a zero-mean stationary (in time) random process with autocorrelation function

$$\langle n(t_1,\boldsymbol{y}_1)n(t_2,\boldsymbol{y}_2)\rangle = F(t_2-t_1)K(\boldsymbol{y}_1)\delta(\boldsymbol{y}_1-\boldsymbol{y}_2). \tag{2}$$

Here $\langle\cdot\rangle$ stands for statistical average with respect to the distribution of the noise sources. For simplicity we will consider that the process n has Gaussian statistics.

The time distribution of the noise sources is characterized by the correlation function $F(t_2-t_1)$, which is a function of t_2-t_1 only by time stationarity. The Fourier transform $\hat{F}(\omega)$ of the time correlation function $F(t)$ is a nonnegative real-valued function proportional to the power spectral density of the sources:

$$\hat{F}(\omega) = \int F(t)e^{i\omega t}\mathrm{d}t. \tag{3}$$

The spatial distribution of the noise sources is characterized by the autocovariance function $\delta(\boldsymbol{y}_1-\boldsymbol{y}_2)K(\boldsymbol{y}_1)$. The process n is delta-correlated in space and K characterizes the spatial support of the sources. It is possible to consider a more general form for the spatial covariance function. This requires the use of semiclassical analysis, but the results do not change qualitatively [1].

B. Statistical stability of the cross correlation function

The empirical cross correlation of the signals recorded at $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ for an integration time T is

$$C_T(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{1}{T}\int_0^T u(t,\boldsymbol{x}_1)u(t+\tau,\boldsymbol{x}_2)\mathrm{d}t. \tag{4}$$

It is a statistically stable quantity, in the sense that for a large integration time T, the empirical cross correlation C_T is independent of the realization of the noise sources. This is stated in the following proposition [8].

Proposition 2.1: 1. The expectation of the empirical cross correlation C_T (with respect to the distribution of the sources) is independent of T:

$$\langle C_T(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2)\rangle = C^{(1)}(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2), \tag{5}$$

where the statistical cross correlation $C^{(1)}$ is given by

$$C^{(1)}(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{1}{2\pi}\iint \mathrm{d}\boldsymbol{y}\mathrm{d}\omega\hat{F}(\omega)K(\boldsymbol{y}) \times\overline{\hat{G}}(\omega,\boldsymbol{x}_1,\boldsymbol{y})\hat{G}(\omega,\boldsymbol{x}_2,\boldsymbol{y})e^{-i\omega\tau}, \tag{6}$$

and $\hat{G}(\omega,\boldsymbol{x},\boldsymbol{y})$ is the time-harmonic Green's function.

2. The empirical cross correlation C_T is a self-averaging quantity:

$$C_T(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2) \overset{T\to\infty}{\longrightarrow} C^{(1)}(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2), \tag{7}$$

in probability with respect to the distribution of the sources.

C. Travel time estimation

The travel time or even the full Green's function between two sensors in an inhomogeneous medium can be estimated using the cross correlation of noisy signals recorded by the sensors. The basic result is that, when the support of the random noise sources extends over all space, then the τ-derivative of the cross correlation of the recorded signals is

978-1-4244-5180-7/09/$26.00 ©2009 IEEE

the symmetrized Green's function between the sensors [10]. The singular components of the cross correlation are therefore related to the travel times between sensors. This can be shown using the Helmholtz-Kirchhoff theorem [14], [8]. This is also true with spatially localized noise source distributions provided the waves propagate within an ergodic cavity [7], [1]. At the physical level this result can be generalized to open and closed environments provided that the recorded signals are equipartitioned [15], [12]. In an open environment this means that the recorded signals are an uncorrelated and isotropic superposition of plane waves of all directions. In a closed environment it means that the recorded signals are superpositions of normal modes with random amplitudes that are statistically uncorrelated and identically distributed.

Unfortunately in many applications the noise source distribution is spatially limited and the recorded signals are not equipartitioned. As a result, the waves recorded by the sensors are dominated by the flux coming from the direction of the noise sources, which affects the quality of the estimate for the Green's function. It is good when the line between the sensors is along the direction of the energy flux and bad when it is perpendicular to it [13]. This can be explained using a stationary phase analysis [8]. Travel time estimation is, therefore, not always possible. It is, however, possible to enhance the quality of travel time estimates by exploiting the enhanced directional diversity provided by the multiple scattering of waves in a scattering medium [5], [8].

D. Passive sensor imaging

Our main focus in this paper is passive sensor imaging. Consider an array of sensors located at $(\boldsymbol{x}_j)_{j=1,\dots,N}$ and small reflectors located at $(\boldsymbol{z}_{\mathrm{r},j})_{j=1,\dots,N_r}$. In imaging we want to estimate the locations of the reflectors from the signals recorded by the sensors. In *active* sensor imaging, the sensors of the array can be used as emitters and as receivers. It is then possible to measure the signal $(P(\boldsymbol{x}_j,\boldsymbol{x}_l,t))_{t\in\mathbb{R}}$ recorded by the lth sensor when the jth sensor emits a short impulse. When the impulse response matrix $(P(\boldsymbol{x}_j,\boldsymbol{x}_l,t))_{j,l=1,\dots,N,t\in\mathbb{R}}$ of the sensor array is known, even partially, then the usual migration techniques [2], [6] that backpropagate the impulse responses numerically in a fictitious medium give estimates of the locations of the reflectors. From this description it seems that knowing the impulse response matrix requires an active, broadband sensor array. However, in *passive* sensor imaging, the sensors of the array can be used only as receivers, and they record the signals $(u(t,\boldsymbol{x}_j))_{j=1,\dots,N,t\in\mathbb{R}}$ emitted by ambient noise sources. It turns out that the impulse response matrix of a passive sensor array can be estimated from the matrix of cross correlations of the recorded noisy signals $(C(\tau,\boldsymbol{x}_j,\boldsymbol{x}_l))_{j,l=1,\dots,N,\tau\in\mathbb{R}}$ (as in (4)). The main idea is that the cross correlation between two sensors has peaks at time lags corresponding to travel times between the sensors and the reflectors. It is therefore possible to image the reflectors by backpropagating the cross correlations.

In order to image the reflectors, we assume that we know the travel times between the sensors and points in the search region around the reflectors to be imaged. If in particular the medium is homogeneous then $\tau(\boldsymbol{x},\boldsymbol{y}) = |\boldsymbol{x}-\boldsymbol{y}|/c_0$. We also assume that data sets $\{C_{\mathrm{r}}(\tau,\boldsymbol{x}_j,\boldsymbol{x}_l)\}_{j,l=1,\dots,N}$ and $\{C(\tau,\boldsymbol{x}_j,\boldsymbol{x}_l)\}_{j,l=1,\dots,N}$, with and without the reflectors, respectively, are available so that we can compute the differential cross correlations $\{C_{\mathrm{r}}-C\}$ and migrate them. The primary data set $\{C_{\mathrm{r}}\}$ cannot be used directly for imaging because peaks in the cross correlations due to the reflectors are weak compared both to the peaks of the direct waves and to the non-singular components due to the directional energy flux [8].

E. Small decoherence time hypothesis

From now on we assume that the decoherence time of the noise sources is much smaller than the typical travel time that we want to estimate (i.e., the travel time from the reflector(s) to the sensor array). If we denote by ε the (small) ratio of these two time scales, then we can write the time correlation function F_ε in the form

$$F_\varepsilon(t_2-t_1) = F\Big(\frac{t_2-t_1}{\varepsilon}\Big), \tag{8}$$

where t_1 and t_2 are scaled relative to typical travel times. The hypothesis $\varepsilon \ll 1$ is both natural and useful:

1) In experimental situations noise records are first band-filtered and then cross correlated [11]. If the central frequency ω_0 of the filter is high enough so that the corresponding wavelength λ_0 is much smaller than the distance d from the sensor array to the reflector(s), then we have $\varepsilon = \lambda_0/d \ll 1$. The resolution is limited by the wavelength (in fact, by the bandwidth), so that it is necessary to assume that $\varepsilon \ll 1$ in order to get some resolution.

2) The Fourier transform of the time correlation function of the sources has the form $\hat{F}^\varepsilon(\omega) = \varepsilon\hat{F}(\varepsilon\omega)$, so that (after the change of variables $\omega \to \omega/\varepsilon$) the statistical cross correlation (6) involves a product of Green's functions evaluated at high frequencies ω/ε. Using a geometric optics approximation of the Green's function and a stationary phase analysis allows us to study the cross correlations in the regime $\varepsilon \ll 1$.

III. Passive sensor imaging of a reflector

In this section we show that it is possible to image reflectors by cross correlations of signal amplitudes generated by ambient noise sources and recorded by passive sensors.

A. The background Green's function

For a background medium with smoothly varying propagation speed $c(\boldsymbol{x})$, the high-frequency asymptotics of the time-harmonic Green's function $\hat{G}$ of the background medium is of the form

$$\hat{G}\Big(\frac{\omega}{\varepsilon},\boldsymbol{x},\boldsymbol{y}\Big) = a(\boldsymbol{x},\boldsymbol{y})\exp\Big(i\frac{\omega}{\varepsilon}\tau(\boldsymbol{x},\boldsymbol{y})\Big). \tag{9}$$

Here the coefficients $a(\boldsymbol{x},\boldsymbol{y})$ and $\tau(\boldsymbol{x},\boldsymbol{y})$ are smooth except at $\boldsymbol{x}=\boldsymbol{y}$. The amplitude $a(\boldsymbol{x},\boldsymbol{y})$ satisfies a transport equation and the travel time $\tau(\boldsymbol{x},\boldsymbol{y})$ satisfies the eikonal equation. Substituting the expression (9) of the Green's function into

(6), we obtain an expression of the statistical cross correlation in the form of a multiple integral with a smooth amplitude and a rapid phase. The stationary phase theorem therefore appears as a natural tool for the study of the cross correlation. As shown in [8] it is possible to analyze travel time estimation by cross correlation in order to identify the conditions under which the cross correlation has singular components at plus or minus the travel time between the sensors. The analysis can also be carried out in the presence of reflectors buried in the medium which generate reflected rays.

B. The cross correlation in the presence of a reflector

We carry out the analysis when the background medium is smoothly varying with background speed $c(\boldsymbol{x})$ and there is a point reflector at $\boldsymbol{z}_{\rm r}$. Since we assume that the reflector is weak and small, we can use the Born approximation for the Green's function:

$$\hat{G}_{\rm r}\big(\omega,\boldsymbol{x},\boldsymbol{y}\big)=\hat{G}\big(\omega,\boldsymbol{x},\boldsymbol{y}\big)+\frac{\omega^2}{c_0^2}\sigma_{\rm r}l_{\rm r}^3\hat{G}\big(\omega,\boldsymbol{x},\boldsymbol{z}_{\rm r}\big)\hat{G}\big(\omega,\boldsymbol{z}_{\rm r},\boldsymbol{y}\big).$$

Here $\hat{G}$ is the Green's function of the background medium, that is, in the absence of reflector, $\sigma_{\rm r}$ is the reflectivity of the reflector, and $l_{\rm r}^3$ is the reflector volume. The statistical differential cross correlation is given by

$$\Delta C^{(1)}(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2)=C_{\rm r}^{(1)}(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2)-C^{(1)}(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2),\quad(10)$$

where $C_{\rm r}^{(1)}$ is the statistical cross correlation in the presence of the reflector (i.e. (6) with the full Green's function $\hat{G}_{\rm r}$) and $C^{(1)}$ is the statistical cross correlation in the absence of the reflector (i.e. (6) with the background Green's function $\hat{G}$). The differential cross correlation technique removes the contributions of the direct waves so that the small singular components of the reflected waves can be observed. The next proposition gives the necessary and sufficient conditions for the existence of singular components in the differential cross correlation. Using stationary phase analysis we can prove the following proposition [8].

Proposition 3.1: When there is a point reflector at $\boldsymbol{z}_{\rm r}$, then the differential cross correlation has different types of singular components in the asymptotic framework $\varepsilon\to 0$. These singular components exist provided special configurations of ray segments exist, as described in Figure 1. More precisely:

(a) If the ray going through $\boldsymbol{x}_1$ and $\boldsymbol{z}_{\rm r}$ extends into the source region and if $\boldsymbol{x}_1$ is between $\boldsymbol{z}_{\rm r}$ and the sources, then there is a singular component at $\tau=\tau(\boldsymbol{x}_1,\boldsymbol{z}_{\rm r})+\tau(\boldsymbol{x}_2,\boldsymbol{z}_{\rm r})$.

(b) If the ray going through $\boldsymbol{x}_2$ and $\boldsymbol{z}_{\rm r}$ extends into the source region and if $\boldsymbol{x}_2$ is between $\boldsymbol{z}_{\rm r}$ and the sources, then there is a singular component at $\tau=-\tau(\boldsymbol{x}_1,\boldsymbol{z}_{\rm r})-\tau(\boldsymbol{x}_2,\boldsymbol{z}_{\rm r})$.

(c) If the ray going through $\boldsymbol{x}_2$ and $\boldsymbol{z}_{\rm r}$ extends into the source region and if $\boldsymbol{z}_{\rm r}$ is between $\boldsymbol{x}_2$ and the sources, then there is a singular component at $\tau=\tau(\boldsymbol{x}_2,\boldsymbol{z}_{\rm r})-\tau(\boldsymbol{x}_1,\boldsymbol{z}_{\rm r})$.

(d) If the ray going through $\boldsymbol{x}_1$ and $\boldsymbol{z}_{\rm r}$ extends into the source region and if $\boldsymbol{z}_{\rm r}$ is between $\boldsymbol{x}_1$ and the sources, then there is a singular component at $\tau=\tau(\boldsymbol{x}_2,\boldsymbol{z}_{\rm r})-\tau(\boldsymbol{x}_1,\boldsymbol{z}_{\rm r})$.

Proposition 3.1 allows us to identify the singular components of the differential cross correlations and it makes clear

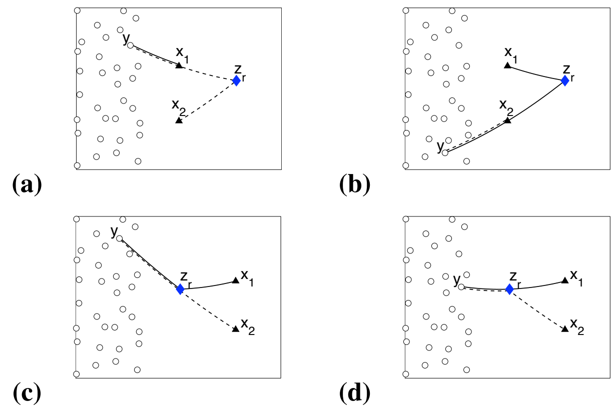


Fig. 1. Configurations of ray segments that contribute to the singular component of the differential cross correlation of signals between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. The circles are noise sources, the triangles are sensors, and the diamond is the reflector. (a-b) are daylight illumination configurations (the sensors are between the noise sources and the reflector), (c-d) are backlight illumination configurations (the reflector is between the noise sources and the sensors).

which is the appropriate imaging functional that should be used to migrate the cross correlations. This functional depends on the illumination configuration. There are two main types of configurations of sources, sensors, and reflectors.

1) The noise sources are spatially localized and the sensors are between the sources and the reflectors (cases *a-b* in Proposition 3.1 and Figure 1). We call this the daylight configuration. In a daylight configuration the singular components of the differential cross correlation are concentrated at (plus or minus) the total travel time $\tau(\boldsymbol{x}_2,\boldsymbol{z}_{\rm r})+\tau(\boldsymbol{x}_1,\boldsymbol{z}_{\rm r})$.

2) The noise sources are spatially localized and the reflectors are between the sources and the sensors (cases *c-d* in Proposition 3.1 and Figure 1). We call this the backlight configuration, in analogy with photography. In a backlight configuration the singular components of the differential cross correlation are concentrated at the difference travel time $\tau(\boldsymbol{x}_2,\boldsymbol{z}_{\rm r})-\tau(\boldsymbol{x}_1,\boldsymbol{z}_{\rm r})$.

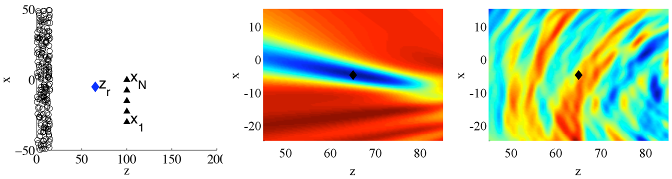
C. Migration imaging of cross correlations

First, we consider migration imaging with daylight illumination. The imaging functional at a search point $\boldsymbol{z}^S$ is the *daylight imaging functional*

$$\mathcal{I}^{\rm D}(\boldsymbol{z}^S)=\sum_{j,l=1}^{N}\Delta C\big(\tau(\boldsymbol{z}^S,\boldsymbol{x}_l)+\tau(\boldsymbol{z}^S,\boldsymbol{x}_j),\boldsymbol{x}_j,\boldsymbol{x}_l\big)+\Delta C\big(-\tau(\boldsymbol{z}^S,\boldsymbol{x}_l)-\tau(\boldsymbol{z}^S,\boldsymbol{x}_j),\boldsymbol{x}_j,\boldsymbol{x}_l\big).\quad(11)$$

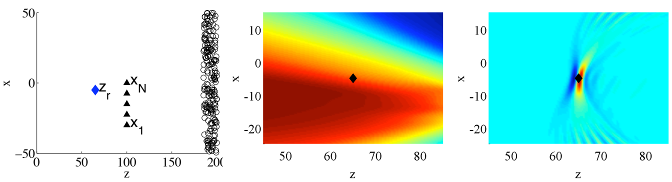
This functional uses the positive and negative parts of the cross correlations, which correspond to the causal and anti-causal Green's functions. It is a consequence of Proposition 3.1 that the matched filter should be chosen at (plus or minus) the sum of the travel times $\tau(\boldsymbol{z}^S,\boldsymbol{x}_l)+\tau(\boldsymbol{z}^S,\boldsymbol{x}_j)$. It is shown there that the singular component of $\Delta C(\tau,\boldsymbol{x}_j,\boldsymbol{x}_l)$ is at $\tau=\pm[\tau(\boldsymbol{z}_{\rm r},\boldsymbol{x}_l)+\tau(\boldsymbol{z}_{\rm r},\boldsymbol{x}_j)]$. By using an analogy with Kirchhoff migration it can be shown that

1) the cross range resolution of the daylight imaging functional for a linear sensor array with aperture a is given by



a) Configuration **b)** $\mathcal{I}^{\mathrm{B}}$ **c)** $\mathcal{I}^{\mathrm{D}}$

Fig. 2. Passive sensor imaging using the differential cross correlation technique in a homogeneous medium. The backlight illumination configuration is plotted in Figure a: the circles are the noise sources, the triangles are the sensors, and the diamond is the reflector. Figure b plots the image obtained with the backlight imaging functional (12). Figure c plots the image obtained with the daylight imaging functional (11).



a) Configuration **b)** $\mathcal{I}^{\mathrm{B}}$ **c)** $\mathcal{I}^{\mathrm{D}}$

Fig. 3. Passive sensor imaging using the differential cross correlation technique in a homogeneous medium. The daylight illumination configuration is plotted in Figure a: the circles are the noise sources, the triangles are the sensors, and the diamond is the reflector. Figure b plots the image obtained with the backlight imaging functional (12). Figure c plots the image obtained with the daylight imaging functional (11).

$\lambda_0 a/d(A,R)$. Here $d(A,R)$ is the distance between the sensor array and the reflectors and λ_0 is the carrier wavelength.
2) The range resolution is equal to $c_0 B^{-1}$ where c_0 is the background velocity and B is the bandwidth.

Next, we consider migration imaging with backlight illumination. The imaging functional at a search point $\boldsymbol{z}^S$ is the *backlight imaging functional*

$$\mathcal{I}^{\mathrm{B}}(\boldsymbol{z}^S) = \sum_{j,l=1}^{N} \Delta C(\tau(\boldsymbol{z}^S, \boldsymbol{x}_l) - \tau(\boldsymbol{z}^S, \boldsymbol{x}_j), \boldsymbol{x}_j, \boldsymbol{x}_l). \quad (12)$$

The sign of the travel time in the argument of the imaging functional is determined by Proposition 3.1. It is shown there that the singular component of $\Delta C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$ is at $\tau = \tau(\boldsymbol{z}_{\mathrm{r}}, \boldsymbol{x}_l) - \tau(\boldsymbol{z}_{\mathrm{r}}, \boldsymbol{x}_j)$. By using an analogy with the incoherent interferometric imaging functional [4], it can be shown that the backlight imaging functional does not provide any range resolution.

D. Numerical simulations

The numerical simulations presented in this paper evaluate the statistical cross correlations $C_{\mathrm{r}}^{(1)}$ and $C^{(1)}$ for different configurations of noise sources, reflector, and scatterers. The statistical cross correlation is what is obtained with the empirical cross correlation C_T for an infinitely large integration time T. The statistical stability (i.e. the fluctuations of C_T with respect to its statistical average for finite T) has been studied in detail theoretically and numerically in [8]. It is not the limiting factor in this type of problems as long as the recording time window can be taken arbitrarily large.

We consider a three-dimensional homogeneous background medium with velocity $c_0 = 1$. We compute the image in the plane (x, z) and use the homogeneous background Green's function (9). The random sources are a collection of 100 randomly located point sources in a layer of size 100×15 with power spectral density $\hat{F}(\omega) = \exp(-\omega^2)$. We consider a point reflector at position $(-5, 60)$ and 5 sensors located at $(-37.5 + 7.5j, 100)$, $j = 1, \ldots, 5$.

In Figure 2 we consider a backlight illumination configuration. We apply both the backlight imaging functional (12) and the daylight imaging functional (11). As predicted by the theory, the backlight imaging functional gives a good cross-range resolution of the target but a very poor range resolution. The daylight imaging functional is not efficient.

In Figure 3 we consider a daylight illumination configuration. We apply both the backlight imaging functional (12) and the daylight imaging functional (11). As predicted by the theory, the daylight imaging functional gives good cross-range and range resolutions of the target. The backlight imaging functional is not efficient.

References

[1] C. Bardos, J. Garnier, and G. Papanicolaou, *Identification of Green's functions singularities by cross correlation of noisy signals*, Inverse Problems, 24 (2008), 015011.

[2] B. L. Biondi, *3D Seismic Imaging*, no. 14 in Investigations in Geophysics, Society of Exploration Geophysics, Tulsa, 2006.

[3] N. Bleistein, J. K. Cohen, and J. W. Stockwell Jr, *Mathematics of multidimensional seismic imaging, migration, and inversion*, Springer Verlag, New York, 2001.

[4] L. Borcea, G. Papanicolaou, and C. Tsogka, *Theory and applications of time reversal and interferometric imaging*, Inverse Problems, 19 (2003), pp. S134-S164.

[5] M. Campillo and L. Stehly, *Using coda waves extracted from microseisms to construct direct arrivals*, Eos Trans. AGU, 88(52) (2007), Fall Meet. Suppl., Abstract S51D-07.

[6] J. F. Claerbout, *Imaging the Earth's interior*, Blackwell Scientific Publications, Palo Alto, 1985.

[7] Y. Colin de Verdière, *Mathematical models for passive imaging. I: General background*, arXiv:math-ph/0610043v1 (2006).

[8] J. Garnier and G. Papanicolaou, *Passive sensor imaging using cross correlations of noisy signals in a scattering medium*, SIAM J. Imaging Sciences, 2 (2009), pp. 396-437.

[9] P. Gouédard, L. Stehly, F. Brenguier, M. Campillo, Y. Colin de Verdière, E. Larose, L. Margerin, P. Roux, F. J. Sanchez-Sesma, N. M. Shapiro, and R. L. Weaver, *Cross-correlation of random fields: mathematical approach and applications*, Geophysical Prospecting, 56 (2008), pp. 375-393.

[10] P. Roux, K. G. Sabra, W. A. Kuperman, and A. Roux *Ambient noise cross correlation in free space: Theoretical approach*, J. Acoust. Soc. Am., 117 (2005), pp. 79-84.

[11] N. M. Shapiro, M. Campillo, L. Stehly, and M. H. Ritzwoller, *High-resolution surface wave tomography from ambient noise*, Science, 307 (2005), pp. 1615-1618.

[12] R. Snieder, *Extracting the Green's function from the correlation of coda waves: A derivation based on stationary phase*, Phys. Rev. E, 69 (2004), 046610.

[13] L. Stehly, M. Campillo, and N. M. Shapiro, *A study of the seismic noise from its long-range correlation properties*, Geophys. Res. Lett., 111 (2006), B10306.

[14] K. Wapenaar and J. Fokkema, *Green's function representations for seismic interferometry*, Geophysics, 71 (2006), pp. SI33-SI46.

[15] R. Weaver and O. I. Lobkis, *Ultrasonics without a source: Thermal fluctuation correlations at MHz frequencies*, Phys. Rev. Lett., 87 (2001), 134301.